An augmented Becker–DeGroot–Marschak mechanism for transaction cycles

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Abstract
We introduce an augmented Becker–DeGroot–Marschak mechanism for the revelation of willingness-to-accept and willingness-to-pay in transaction cycles. The mechanism can be used to test for a behavioral anomaly.

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Compensating variation
Equivalent variation
Endowment effect
Welfare change
Willingness-to-accept
Willingness-to-pay

1. Introduction

The well-known Becker–DeGroot–Marschak (BDM) mechanism (Becker et al., 1964) is widely used in experimental economics to measure an agent’s willingness-to-pay (WTP) or willingness-to-accept (WTA) for a lottery (Lichtenstein and Slovic, 1971; Roth, 1995). It has also been used to test for significant disparities between WTA and WTP, which some have argued can be explained by loss aversion (Thaler, 1980; Knetsch, 1989; Kahneman et al., 1990). Weber (2010) shows that the existence of endowment effects is unsurprising from a normative viewpoint, and that to test for a behavioral anomaly, proper income compensation along a full transaction cycle is required. The ideal measurement would proceed as follows. Given the agent’s income \( y \), in one direction of such a full transaction cycle, an agent’s WTP \( WTP(y) \) for obtaining a nonmarket good is elicited, then fully extracted. Subsequently, the agent’s income-compensated WTA \( WTA(y - WTP(y)) \) for returning that same nonmarket good is elicited and compared to the initially reported WTP in order to test the normative identity \( WTP(y) = WTA(y - WTP(y)) \). Income compensation means that as part of the first transaction in the cycle, the agent’s original income \( y \) is reduced by \( WTP(y) \), that is, the agent needs to pay an amount equal to her full value of the item. If the agent possesses the good initially, the transaction cycle can be reversed to symmetrically test the normative identity \( WTA(y) = WTP(y + WTA(y)) \).

In practice, the income \( y \) and the valuations are part of the agent’s private information. The fundamental impossibility of full rent extraction in the presence of asymmetric information implies that it is not possible to actually implement the required income compensation as part of the first transfer. In both portions of the transaction cycle, a positive information rent is required (at least in expectation) to encourage the agent to report her valuations truthfully. Moreover, what links incentives across the two transfers is that the prospect of a positive information rent for truthfully revealing private information in the course of the second (return) transfer tends to distort the agent’s report to the principal during the first transfer, because she becomes overly eager to reach the second transfer. Thus, when income compensation is required (and anticipated by the agent) one cannot use the standard BDM mechanism for both portions of a transaction cycle. In this paper, we propose an augmented BDM mechanism that produces testable hypotheses for the (income-compensated) endowment effect as a behavioral anomaly. The augmented BDM mechanism can be adjusted so as to render the distortion from full revelation along the transaction cycle arbitrarily small.
2. Preliminaries

WTA and WTP directly correspond to the Hicksian welfare measures of equivalent variation (E) and compensating variation (C), respectively, that are used in demand theory to monetize the effect of a given state change (from “0” to “1”) on a consumer from an ex-ante or an ex-post perspective (Hicks, 1939). Given an income y and an agent’s increasing indirect utility functions v₀(y) before the state change and v₁(y) after the state change, the agent’s WTA and WTP for the change are implicitly determined by v₁(y) = v₀(y + E(y)) and v₀(y) = v₁(y − C(y)), respectively.² This leads to income-compensated normative identities between the Hicksian welfare measures (Weber, 2003, 2010).

\[
C(y) = E(y - C(y)),
\]

(1)

\[
E(y) = C(y + E(y)),
\]

(2)
as a function of y. A violation of these identities can be tested empirically using the following procedure.

3. Mechanism

Assume that the agent’s income y is unknown to the experimenter,³ who confronts the agent with a possible transition between two states termed “0” and “1”. We assume that the transition from state 0 to state 1 is desirable. The classical setup is that in state 1 the agent possesses a certain valuable nonmarket good such as an idiosyncratic coffee mug, access to a public park, a better grade, and so forth. We make no assumption as to the precise nature of this state change, other than that the agent is willing to pay a nonnegative amount of her income for it.⁴ The standard BDM mechanism achieves truthful revelation of a private value by asking the agent to announce her WTP (resp. WTA) for the transition from 0 to 1 (resp. from 1 to 0), given that the actual payment amount is the realization of a real-valued random variable on a sufficiently large support,⁵ and that a transaction takes place only if ex post the agent’s bid exceeds (resp. is less than) the realized transfer. Under this mechanism the agent expects a positive information rent in return for truthful revelation.

The following augmented BDM mechanism allows for two consecutive nontrivial state transitions (a ‘transaction cycle’), either from 0 to 1 to 0 (termed mechanism \( \mathcal{M} \)), or from 1 to 0 to 1 (termed mechanism \( \mathcal{M} \)). We describe both versions of the mechanism in turn. In what follows we sometimes omit the explicit dependence on the fixed income level y for convenience.

\[\text{Mechanism } \mathcal{M} \text{ (Initial State: 0)}\]

Step 1. Solicit the agent’s WTP \( \beta \) for switching from state 0 to state 1 using a standard BDM mechanism. The actual transfer \( t \) and the state transition itself depend on the realization \( t \) of a random variable \( \tilde{t} \). If \( t \leq \beta \), then the state transition will take place and \( \tau = t \). If \( t > \beta \), the state transition does not take place and \( \tau = 0 \). The realization \( t \) is determined but not yet revealed to the agent.

Step 2. Tell the agent that a realization of the random variable \( \tilde{t} \) has been determined. Then ask the agent to assume that the state transition from state 0 to state 1 takes place, and eliciting her WTA \( \alpha(t) \) to transition from state 1 to state 0 as a function of all possible realizations \( t \in [0, \beta] \). As in Step 1, the actual transfer \( \sigma \) depends on the realization \( s \) of a random variable \( \hat{s} \). If \( s \geq \alpha(t) \), then the state transition from 1 to 0 will take place, and the agent is paid the amount \( \sigma = s \). If \( s < \alpha(t) \), then the system remains in state 1, and the agent obtains \( \sigma = 0 \). The realization \( s \) is determined but not yet revealed to the agent.

Step 3. Both \( t \) and \( s \) are revealed to the agent. If \( t > \beta \), the agent obtains nothing, and no state transition takes place. If \( t \leq \beta \) and \( s < \alpha(t) \), then the agent pays \( t \), and the state transitions from 0 to 1. Lastly, if \( t \leq \beta \) and \( s \geq \alpha(t) \), the agent obtains the net transfer \( s - t \), and no state transition takes place.

\[\text{Remark 1.} \text{ The reason for eliciting the function } \alpha(t) \text{ for } t \in [0, \beta] \text{ instead of just the point } \alpha(\beta) \text{ is to enable a well-defined transition from state 1 to state 0 contingent on } s \geq \alpha(t). \]

6 In practice, one may use an n-point discretization to obtain \( \alpha(s) \) for \( t_k = k/n \) for \( k \in \{1, \ldots, n\} \) and then determine the net transfer for the return transition using linear interpolation.

\[\text{Mechanism } \hat{\mathcal{M}} \text{ (Initial State: 1)}\]

Step 1. Solicit the agent’s WTA \( \hat{\alpha} \) for switching from state 1 to state 0. The actual transfer \( \tau \) and state transition depend on the realization \( s \) of a random variable \( \hat{s} \). If \( s \geq \hat{\alpha} \), then the state transition will take place and \( \sigma = s \). If \( s < \hat{\alpha} \), the state transition does not take place and \( \sigma = 0 \). The realization \( s \) is determined but not yet revealed to the agent.

Step 2. Tell the agent that a realization of the random variable \( \hat{s} \) has been determined. Then ask the agent to assume the state transition takes place, and elicit her WTP \( \hat{\beta}(s) \) as a function of all possible realizations \( s \in [\hat{\alpha}, \hat{\alpha}] \), where \( \hat{\alpha} \) is chosen as in Remark 1. As in Step 1, the actual transfer \( \tau \) depends on the realization \( t \) of a random variable \( \tilde{t} \). If \( t \leq \beta(s) \), then the state transition from 0 to 1 will take place and the agent pays the amount \( \tau = t \). If \( t > \beta(s) \), then the system remains in state 0 and the agent pays \( \tau = 0 \). The realization \( t \) is determined but not yet revealed to the agent.

Step 3. Both \( s \) and \( t \) are revealed to the agent. If \( s < \hat{\alpha} \), the agent obtains nothing and no state transition takes place. If \( s \geq \hat{\alpha} \) and \( t > \beta(s) \), then the agent obtains \( s \) and the state transitions from 1 to 0. Lastly, if \( s \geq \hat{\alpha} \) and \( t \leq \beta(s) \), the agent obtains the net transfer \( s - t \) and no state transition takes place.

\[\text{Remark 2.} \text{ The supports of the random variables } \hat{s} \text{ and } \tilde{t} \text{ are chosen as in Remark 1. Analogous to mechanism } \mathcal{M} \text{, eliciting the function } \hat{\pi}(s) \text{ for all } s \in [\hat{\alpha}, \hat{\alpha}] \text{ guarantees the correct contingent transfer for the possible return state transition (see also Footnote 6 regarding a discretization of the agent’s response in practice).} \]
4. Main result

Under the assumption of common knowledge between the agent and the mechanism operator about all procedures described in Section 3, both \( \mathcal{M} \) and \( \hat{\mathcal{M}} \) produce truthful revelations by the agent on the return transition, as a function of possible transfer realizations in the first state transition. When submitting a bid for the return transition, the agent expects a positive information rent. This information rent tends to exaggerate the agent’s motivation to effect the first state transition, thereby resulting in overbidding realizations in the first state transition. When submitting a bid for the agent on the return transition, as a function of possible transfer realizations in the first state transition. Since \( \pi_1(y, t) \equiv \max \{0, v_0(y, t - \hat{s}) - v_1(y, t)\} \), the return transition.

This condition is similar in the standard BDM mechanism and can be motivated empirically testable inequalities. Therefore, because \( \hat{\beta} \equiv \hat{\beta}(\hat{\alpha}) \) with \( F \equiv \hat{\beta}(\hat{\alpha}) \), so it is optimal for the agent to offer \( \hat{\alpha} \) such that \( \hat{\pi}_2(\hat{\alpha}, \hat{\alpha}) = v_1(y) \). But since \( \hat{\pi}_2(y, s) \geq v_0(y, s) \) for all relevant \( s \), we obtain that \( \hat{\alpha} \leq E(y) \).

\[ \text{Fig. 1 provides some intuition. Under } \mathcal{M}, \text{ given any realization } t \in [0, \beta], \text{ at the beginning of Step 2 the agent can expect a positive information rent (over her default payoff } v_1(y, t) \text{ without private information),} \]

\[ \pi_2(y, t) = \pi_2^*(y, t) - v_1(y, t) \]

\[ = E\left(\left[v_0(y, t - \hat{s}) - v_1(y, t - \hat{s})\right]\right) \]

\[ = E\left[\max\{0, v_0(y, t - \hat{s}) - v_1(y, t - \hat{s})\}\right]. \]

Thus, because \( \pi_2^* = v_1 + R_2 \geq v_1 \), her expected utility \( \pi_1(\beta) \) exceeds the standard BDM payoff, \( E\left[v_0(y, \hat{t}) + v_1(y, \hat{t})\right] \), which in turn implies that the agent’s bid \( \beta \) in Step 1 exceeds her WTP \( C(y) \). The reasoning under \( \mathcal{M} \) is similar. Here the agent shaves her reported WTA \( \hat{\alpha} \) for the first transition below her actual WTA \( E(y) \) to increase the probability that it takes place, for she expects \( \hat{\alpha} \equiv \hat{\beta}(\hat{\alpha}) \) a positive information rent \( \hat{R}_2(y, \hat{s}) = \max\{0, v_1(y + s - \hat{t}) - v_0(y, s)\} \) from the return transition.\(^8\) One therefore obtains the relations that define the Step 1 bids for both mechanisms.\(^9\)

**Corollary 1.** The bid \( \beta (\text{resp. } \hat{\alpha}) \) in Step 1 of mechanism \( \mathcal{M} \) (resp. \( \hat{\mathcal{M}} \)) is determined by \( v_0(y) = v_1(y - \hat{\beta}) + R_2(y, \hat{\beta}) \) (resp. \( v_1(y) = v_0(y + \hat{\alpha}) + \hat{R}_2(y, \hat{\alpha}) \)).

The following result compares the Step 1 bids to the corresponding Step 2 bids with full income compensation, which yields empirically testable inequalities.

**Corollary 2.** Let \( (\beta, \alpha(\cdot)) \) be the agent’s response observed under mechanism \( \mathcal{M} \), and let \( (\hat{\alpha}, \hat{\beta}(\cdot)) \) be her response observed under mechanism \( \mathcal{M} \). Then, corresponding to Eqs. (1) and (2), the inequalities

\[ \beta \leq \alpha(\beta), \quad (1') \]

and

\[ \hat{\alpha} \leq \hat{\beta}(\hat{\alpha}), \quad (2') \]

hold.

**Proof.** From the fact that under mechanism \( \mathcal{M} \) the agent reports \( \beta \geq C(y) \), we infer that

\[ \pi_2^* = \pi_2^*(y, \hat{s}), \quad s \geq 0, \text{ submitting a bid of } \beta(s) \equiv C(y) + s \text{ is a dominant strategy for the agent. This strategy is unique because of the strict monotonicity of the indirect utility. In Step 1, the agent faces an expected utility of } \]

\[ \hat{\pi}_2(\hat{\alpha}, \hat{\alpha}) = E\left[v_1(y, t)1_{[t, \beta]} + \pi_2^*(y, \hat{s})1_{[s, \beta]}\right], \]

so it is optimal for the agent to offer \( \hat{\alpha} \) such that \( \hat{\pi}_2(\hat{\alpha}, \hat{\alpha}) = v_1(y) \). But since \( \hat{\pi}_2(y, s) \geq v_0(y, s) \) for all relevant \( s \), we obtain that \( \hat{\alpha} \leq E(y) \).

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\(^7\) This condition is similar in the standard BDM mechanism and can be motivated as follows. If \( \hat{I} \) is distributed on \( \mathcal{R}_I \), according to the distribution function \( F \) with density \( f \), then the maximand becomes

\[ \pi_1(\beta) = \int_0^\beta \pi_1^*(y, t) \mu(dy) + \int_\beta^\infty v_0(y) df(t); \]

the first-order condition yields \( \pi_1^*(y, \beta) = v_0(y) \) as long as \( f(\beta) > 0 \).

\(^8\) In Step 1, the agent expects an information rent \( R_1(y) = E\left[\max\{0, v_1(y - \hat{t}) + R_2(y - \hat{t}, \hat{t})\}1_{[t, \beta]}\right] \) under \( \mathcal{M} \), and \( \hat{R}_1(y) = E\left[\max\{0, v_1(y + s) + \hat{R}_2(y + s, \hat{s})\}1_{[t, \beta]}\right] \) under \( \hat{\mathcal{M}} \).

\(^9\) For a risk-neutral agent with \( v_0(y) = y \) and \( v_1(y) = y + c \) for some constant \( c > 0 \), we have that \( C(y) = E(y) = c \). Under \( \mathcal{M} \), the agent reports \( \beta = c + E[\max\{0, c - y\}] = E\left[\max\{0, c - y\}\right] > C(y) \) and \( \alpha(t) = c \). Under \( \hat{\mathcal{M}} \), she reports \( \hat{\alpha} = c - E\left[\max\{0, c - \hat{t}\}\right] = E\left[\min\{c, \hat{t}\}\right] > E(y) \) and \( \hat{\beta}(\hat{\alpha}) = c = C(y + s) \).
Remark 3. If the possibility of a return transition is unanticipated by the agent, then we obtain truthful revelation in both state transitions and the testable equalities (1′) and (2′) can be replaced by the corresponding equalities. In practical settings, agents typically are not able to predict the course of experiments. Our analysis allows for an agent who knows exactly how the mechanism works, so it can be run repeatedly on the same agent population without affecting the results. In other words, it is not possible to “game” the mechanism.

Remark 4. The information rent from the return transition can be made arbitrarily small by allowing for that transition to take place only with a sufficiently low probability ε > 0, provided it would take place under the original mechanism. The information rents in the corresponding mechanism \( M_\varepsilon \) (resp. \( \hat{M}_\varepsilon \)) becomes \( R_\varepsilon = \varepsilon R_e \) (resp. \( \hat{R}_\varepsilon = \varepsilon R_e \)). Thus, the inequalities (1′) and (2′) tend to the corresponding equalities, or more precisely: \( \lim_{\varepsilon \downarrow 0} \hat{R}_\varepsilon = C(y) \) and \( \lim_{\varepsilon \downarrow 0} \hat{\alpha}_\varepsilon = E(y) \).10

5. Discussion

The agent’s revelations in the augmented BDM mechanism are subject to a one-sided bias, resulting in the inequalities (1′) and (2′). Under \( M \), the basic idea is that the agent, expecting that she may be able to sell if she initially buys, is willing to state a valuation above her true WTP because there is a chance, if she ends up buying the good in Step 1, that she will sell for a price in excess of her true WTA in Step 2 so as to compensate her for the inflated bid in Step 1. The reasoning for the one-sided bias under \( \hat{M} \) is similar.11 A violation of (1′) or (2′) constitutes evidence for a behavioral anomaly. Independent of the initial state, these inequalities predict that the reported WTP must weakly exceed the reported WTA. This is due to the income compensation required in the normative identities (1) and (2). Thus, if the agent’s reported WTA strictly exceeds her reported WTP under either \( M \) or \( \hat{M} \), there is a violation of the normative endowment effect. The interest in the augmented BDM mechanism is therefore to provide a means for testing the robustness of the WTA–WTP disparities with respect to income compensation. It is noteworthy that the inequalities (1′) or (2′) have a “normative direction”, in the sense that they allow the experimenter to falsify the normative relation between Hicksian welfare measures through data that show that WTA exceeds WTP on a transaction cycle with proper income compensation. If these inequalities were reversed, nothing could be concluded from observing such an income-compensated endowment effect. Note also that the standard BDM mechanism cannot be used for testing the normative identities (1) and (2) because full income compensation along the first state transition would take place with probability zero. The augmented BDM mechanism resolves this issue along a transaction cycle by making a standard BDM mechanism for the second transition contingent on the transfer produced by a standard BDM for the first transition, as part of one single mechanism. While only one of \( M \) and \( \hat{M} \) is needed to test for a behavioral anomaly, an experimental setup can employ both versions of the augmented BDM mechanism in order to control for the orientation of the transaction cycle.

References


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10 The same effect can be obtained by choosing distributions for \( \tilde{\varepsilon} \) and \( \hat{\varepsilon} \), respectively, which render the second state transition sufficiently unlikely.

11 As discussed in Remark 4, the bias can be made arbitrarily small.